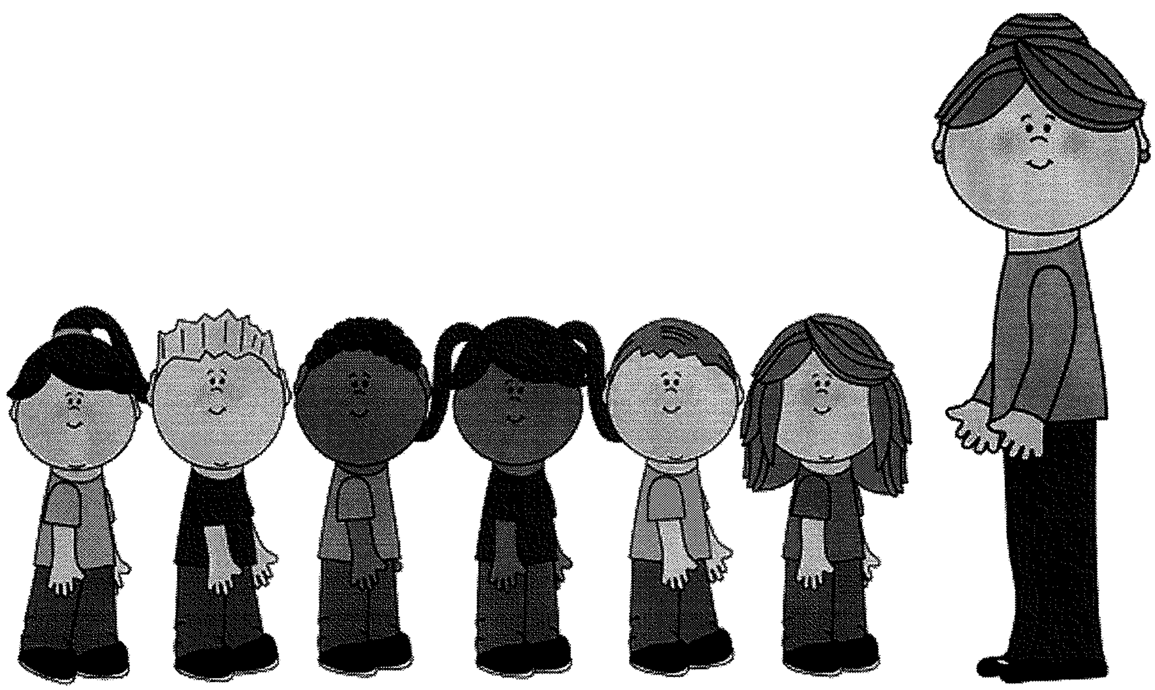
Santee School District

MATHEMATICS PROFESSIONAL DEVELOPMENT

Grade Two



December 6, 2013

MENTAL MATH

During recess there are 98 students on the playground. 87 students join them. How many students are now on the playground?

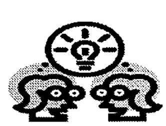
Record how you mentally solved this problem.

Mathematically Productive Teaching Routine
Structuring Student Math-Talk

Purposes
- Support the development of student-to-student interaction that is consistently equitable, status-free, and mathematically productive
- Provide formative assessment information that drives instructional decisions

Student Outcomes
- Equitable, status-free, and mathematically productive student-to-student interaction
- Increased metacognitive skills
- Increased capacity to articulate and clarify their math thinking
- Increased math content knowledge
- Improved Mathematical Habits-of-Mind
- Increased accountability and engagement
- Increased self-efficacy as mathematicians

Structures
When students work in a Mathematicians Dyad, Triad, or Quad, the math-talk:
- <u>Always begins</u> with "Mathematicians Think Time" (i.e., time to think privately) about the task
- <u>Always focuses</u> on each group member's mathematical reasoning, sense making, representations, justifications, and/or generalizations
- <u>Always ends</u> with a discussion of ways their ideas are mathematically the same and/or different
- <u>Always follows</u> a prescribed structure that provides students "practice" with status-free, and mathematically productive student-to-student interaction

LISTEN & COMPARE 	A. Partner #1 explains her/his ideas while the other partner(s) silently listen to understand Partner #1's thinking. B. When the teacher announces, "*Finish your thought and switch roles*," repeat step A for question/task and student backgrounds. C. (for triads and quads) Repeat until all partners have reported.
REVOICE & COMPARE 	A. Partner #1 speaks while the other partner(s) silently listen to understand Partner #1's mathematical thinking. B. When the teacher announces, "*Finish your thought and Partner #X revoice,*" Partner #X carefully revoices Partner #1's ideas without judging, adapting, or commenting about the correctness or sensibility of the ideas. C. Partner #1 clarifies as needed. D. When the teacher announces, "*Rotate Partners*," Partner #2 speaks while the other partner(s) silently listen to understand. E. When the teacher announces, "*Finish your thought and Partner #Y revoice*," Partner #Y carefully revoices Partner #2's ideas. F. Partner #2 clarifies as needed. G. (for triads and quads) Repeat until all partners have revoiced and reported.
INTERPRET & COMPARE 	A. Two partners exchange their written work for a task. During Private Think Time, the partners study each other's work and, without any discussion, try to understand each other's reasoning. B. Partner #1 reports her interpretation of Partner #2's reasoning. C. Partner #2 clarifies. D. Partner #2 reports his interpretation of Partner #1's reasoning. E. Partner #1 clarifies.

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions,

communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Mathematics | Grade 2

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

(1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

(4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Grade 2 Overview

Operations and Algebraic Thinking

- **Represent and solve problems involving addition and subtraction.**
- **Add and subtract within 20.**
- **Work with equal groups of objects to gain foundations for multiplication.**

Number and Operations in Base Ten

- **Understand place value.**
- **Use place value understanding and properties of operations to add and subtract.**

Measurement and Data

- **Measure and estimate lengths in standard units.**
- **Relate addition and subtraction to length.**
- **Work with time and money.**
- **Represent and interpret data.**

Geometry

- **Reason with shapes and their attributes.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 2.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Number and Operations in Base Ten 2.NBT

Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
   a. 100 can be thought of as a bundle of ten tens — called a "hundred."
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2. Count within 1000; skip-count by 5s, 10s, and 100s.
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
9. Explain why addition and subtraction strategies work, using place value and the properties of operations.[3]

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[1]See Glossary, Table 1.
[2]See standard 1.OA.6 for a list of mental strategies.
[3]Explanations may be supported by drawings or objects.

Measurement and Data 2.MD

Measure and estimate lengths in standard units.

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
3. Estimate lengths using units of inches, feet, centimeters, and meters.
4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

Represent and interpret data.

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[4] using information presented in a bar graph.

Geometry 2.G

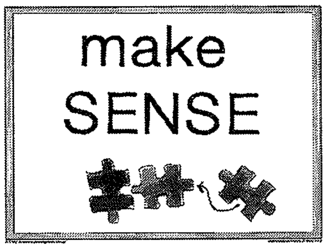
Reason with shapes and their attributes.

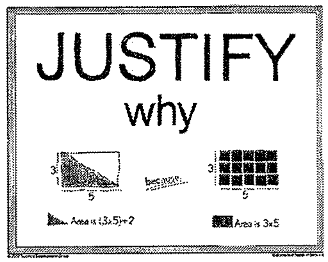
1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[5] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

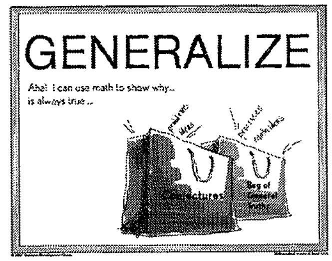
[4]See Glossary, Table 1.
[5]Sizes are compared directly or visually, not compared by measuring.

Name: **Date:**

STUDENT REFLECTION TOOL: MATHEMATICAL HABITS OF MIND



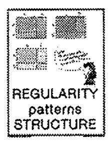
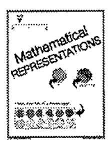
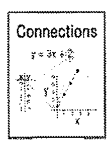
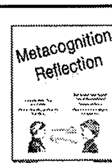
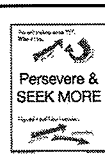




To **make sense** of math ideas and problems, I look for *regularity, patterns, structure, representations, connections, and other math I know*. I *reflect* about my own and others' thinking and *mistakes* and I *persevere* to be sure that ideas and problems make sense.

Math ideas and solutions make sense when I can use *regularity, patterns, structure, representations, connections, and other math I know* to **justify** why the ideas and solutions are always, sometimes, or never true.

I use *regularity, patterns, structure, representations, connections, and other math I know* to make **conjectures** about math ideas I think are always, sometimes, or never true. I create **mathematical generalizations** by justifying why conjectures are valid.

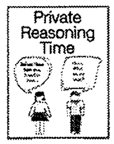
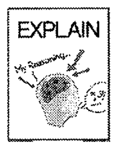
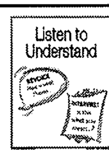
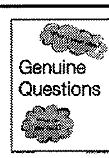
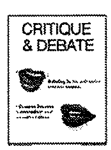
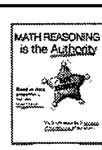
To make **S**ense of math ideas and problems, and to support **C**onjectures, **J**ustifications, and **G**eneralizations:	**S, C, J, G**	**Evidence**
I notice and reason about mathematical REGULARITY in repeated reasoning, PATTERNS, and STRUCTURE (meanings, properties, definitions). 		
I create and reason from MATHEMATICAL REPRESENTATIONS – visual models, graphs, numbers, symbols and equations, and situations. 		
I notice and reason about CONNECTIONS within and across mathematical representations, other math ideas, and everyday life. 		
I explore MISTAKES and STUCK POINTS to start new lines of reasoning and new math learning. 		
I use METACOGNITION and REFLECTION. I think about my math reasoning and disequilibrium – how my thinking is changing and how my ideas compare to other mathematicians' ideas. 		
I PERSEVERE and SEEK MORE. I welcome challenging math problems and ideas, and after I figure something out, I explore new possibilities. 		

Name: **Date:**

STUDENT REFLECTION TOOL: MATHEMATICAL HABITS OF INTERACTION

Use the following scale to rate your use of the Mathematical Habits of Interaction.

0	1	2	3	4
Never/Hardly ever		*Sometimes*		*It's just how I do math!*

When I do math with other mathematicians, we:		Rating	Evidence
Honor each other's right to PRIVATE REASONING TIME before talking about our ideas.			
EXPLAIN how we think and reason mathematically.			
LISTEN TO UNDERSTAND each other's math reasoning about problems, conjectures, justifications, and generalizations.			
Use GENUINE QUESTIONS to inquire about each other's math reasoning about problems, conjectures, justifications, and generalizations.			
Explore MULTIPLE PATHWAYS by applying each other's lines of reasoning.			
COMPARE our math LOGIC and IDEAS to figure out how they are mathematically the same and different.			
CRITIQUE and DEBATE the math logic and truth in each other's reasoning.			
Use MATH REASONING as the AUTHORITY for deciding what is correct and makes sense.			

Hess' Cognitive Rigor Matrix & Curricular Examples: Applying Webb's Depth-of-Knowledge Levels to Bloom's Cognitive Process Dimensions – *M-Sci*

Revised Bloom's Taxonomy	Webb's DOK Level 1 Recall & Reproduction	Webb's DOK Level 2 Skills & Concepts	Webb's DOK Level 3 Strategic Thinking/ Reasoning	Webb's DOK Level 4 Extended Thinking
Remember Retrieve knowledge from long-term memory, recognize, recall, locate, identify	o Recall, observe, & recognize facts, principles, properties o Recall/ identify conversions among representations or numbers (e.g., customary and metric measures)			
Understand Construct meaning, clarify, paraphrase, represent, translate, illustrate, give examples, classify, categorize, summarize, generalize, infer a logical conclusion (such as from examples given), predict, compare/contrast, match like ideas, explain, construct models	o Evaluate an expression o Locate points on a grid or number on number line o Solve a one-step problem o Represent math relationships in words, pictures, or symbols o Read, write, compare decimals in scientific notation	o Specify and explain relationships (e.g., non-examples/examples; cause-effect) o Make and record observations o Explain steps followed o Summarize results or concepts o Make basic inferences or logical predictions from data/observations o Use models /diagrams to represent or explain mathematical concepts o Make and explain estimates	o Use concepts to solve non-routine problems o Explain, generalize, or connect ideas using supporting evidence o Make and justify conjectures o Explain thinking when more than one response is possible o Explain phenomena in terms of concepts	o Relate mathematical or scientific concepts to other content areas, other domains, or other concepts o Develop generalizations of the results obtained and the strategies used (from investigation or readings) and apply them to new problem situations
Apply Carry out or use a procedure in a given situation; carry out (apply to a familiar task), or use (apply) to an unfamiliar task	o Follow simple procedures (recipe-type directions) o Calculate, measure, apply a rule (e.g., rounding) o Apply algorithm or formula (e.g., area, perimeter) o Solve linear equations o Make conversions among representations or numbers, or within and between customary and metric measures	o Select a procedure according to criteria and perform it o Solve routine problem applying multiple concepts or decision points o Retrieve information from a table, graph, or figure and use it solve a problem requiring multiple steps o Translate between tables, graphs, words, and symbolic notations (e.g., graph data from a table) o Construct models given criteria	o Design investigation for a specific purpose or research question o Conduct a designed investigation o Use concepts to solve non-routine problems o Use & show reasoning, planning, and evidence o Translate between problem & symbolic notation when not a direct translation	o Select or devise approach among many alternatives to solve a problem o Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze Break into constituent parts, determine how parts relate, differentiate between relevant-irrelevant, distinguish, focus, select, organize, outline, find coherence, deconstruct	o Retrieve information from a table or graph to answer a question o Identify whether specific information is contained in graphic representations (e.g., table, graph, T-chart, diagram) o Identify a pattern/trend	o Categorize, classify materials, data, figures based on characteristics o Organize or order data o Compare/ contrast figures or data o Select appropriate graph and organize & display data o Interpret data from a simple graph o Extend a pattern	o Compare information within or across data sets or texts o Analyze and draw conclusions from data, citing evidence o Generalize a pattern o Interpret data from complex graph o Analyze similarities/differences between procedures or solutions	o Analyze multiple sources of evidence o analyze complex/abstract themes o Gather, analyze, and evaluate information
Evaluate Make judgments based on criteria, check, detect inconsistencies or fallacies, judge, critique			o Cite evidence and develop a logical argument for concepts or solutions o Describe, compare, and contrast solution methods o Verify reasonableness of results	o Gather, analyze, & evaluate information to draw conclusions o Apply understanding in a novel way, provide argument or justification for the application
Create Reorganize elements into new patterns/structures, generate, hypothesize, design, plan, construct, produce	o Brainstorm ideas, concepts, or perspectives related to a topic	o Generate conjectures or hypotheses based on observations or prior knowledge and experience	o Synthesize information within one data set, source, or text o Formulate an original problem given a situation o Develop a scientific/mathematical model for a complex situation	o Synthesize information across multiple sources or texts o Design a mathematical model to inform and solve a practical or abstract situation

Student Mathematical Discourse Types

Discourse Types	Examples from Case Study
PROCEDURES/FACTS *No evidence of reasoning.* • Short answer to a direct question • Restating facts/statements/rules • Showing or asking for procedures *Uses meanings, definitions, properties, known math ideas to describe reasoning when:* • Explaining ideas and methods • Questioning to clarify • Noticing relationships/connections • **But *doesn't show why* the ideas/methods work**	
JUSTIFICATION *Reasons with meanings of ideas, definitions, math properties, established generalizations to:* • Show why an idea/solution is true • Refute the validity of an idea • Give mathematical defense for an idea that was challenged	
GENERALIZATION *Reasons with math properties, definitions, meanings of ideas, established generalizations, and mathematical relationships as the basis for:* • Making conjectures about what might happen in the general or special cases Or • Justifying a conjecture about what will happen in the general or special cases	

Who invented zero anyway?
Muriel
Grade 2, April

My second graders and I were looking at the hundreds chart set up in a 10-by-10 array. I had imagined leading a discussion toward the idea that moving down one space is actually adding 10. As we got into the discussion, I found that it is, of course, a complicated idea; even if I just tell my students that it works this way, they don't "get it" in any kind of meaningful way. Besides that, several of my students raised a very different idea – an idea about their understanding of zero. Following is a portion of our discussion.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Beth is describing for me – and anyone who is listening – why it works to move down one and actually add ten on this particular hundreds chart.

Beth: See the nines? [*She points to the column of 9, 19, 19, 29, 39 . . .*] This is 0 + 9, 10 + 9, 20 + 9, 30 + 9. The difference between 50 and 60 is 10, so the difference between 59 and 69 is 10.

Quite articulate, I think. Then Beth continues.

Beth: [*She points to 45.*] This has 4 tens and one 5. These [*pointing to the columns with numbers ending in 6 or 7]* all have sixes, sevens. [*Now she points to the 10, 20, 30 column.*] This one has zeros. These aren't quite zero, because . . .

Her voice kind of trails off and my mind is racing. What did she say? "These aren't quite zero"?

Teacher: Because why? [*I am trying desperately to figure out in that moment all of what she is saying and trying to sort out what I might ask next.*]

Beth: Because if zero isn't here [*she points to 60*], then this 6 is only 6. It depends on where it [the zero] is. See, 15. [*She writes 15 on the board.*] This [*points to the 1*] is a 10, not a 1. Ten has everything in it up to 9. The ten section has from 10 to 99. The hundred section has – say it's one hundred and twenty-five [*She write 125 on the board.*] But you don't write it as 100 or else it would look like this. [*She write 100205 on the board.*]

I'm in that place where I find myself spending a lot of time: hearing many rich ideas, and wondering which ideas I should push on. I decide to push on Beth's ideas about the zero thing.

Teacher: You've written 125 separately, 100205. But tell me some more about zero.

Beth: It is sort of zero, but not exactly. [*In 30*] this zero makes it be 30. If this zero weren't there, it'd be 3.

Yessica: I have something to say about how these zeros aren't really zeros.

Attempting to include some other children in the discussion, I restate what Beth has said, about the zero at the end of the number not being quite the same zero as zero by itself. Yessica comes up to the board.

Yessica: [*Writing 07 on the board*] That's 7. [*Now writes 0.7*]. That's 0 point 7. [*And then she writes 70.*] That's 70. Zero represents 7 tens.

I am completely intrigued by these ideas and love the term represents

Teacher: So if this zero [*point to 0 in 70*] represents tens, in this number, 79, does the 9 represent tens?

Beth: Do you know what we really mean? Do you know the real thing?

Lately Beth has been responding to my questions as if I did already know many of the answers I'm asking her to explain to me. I like to make my questioning as authentic as I can, but the fact is that I usually understand the mathematics that I'm asking the children to think through. Today my questions are framed to find out what the children are thinking, to hear their ideas. I know that the 9 in 79 represents 9 ones, but I'm not sure what Beth thinks.

I explain to her that in my math class for adults, we've talked a lot about zero and what it means to different people, and what it's worth, and whether it's odd or even, for example. And I express my honest interest in what second graders think about zero. This is heard by several other people in class and they perk up a bit. The discussion continues.

Yessica: On the calculator there's a 07.

Teacher: And then is 0 worth zero?

Yessica: Yes [*Other children nod in agreement.*]

Teacher: But not in the 70?

Yessica: Right

Brian: For just 7, the zero doesn't have to be there, just the 7.

Lamont: There are two ways to make zero. This is the 7 for the tens and it [the zero] makes 70.

Teacher: What's the other kind of zero?

Lamont: For the ones. [*He write 08.*]

I'm thinking, "Two kinds of zeros? Wow."

Beth: It's like you sort of understand it, but nobody really understands it. Maybe someone will come around and figure it out. And who invented zero anyway?

I laugh and write he question on the board.

Wenona: Yea, and who invented numbers anyway?

I write this down on the board also.

Teacher: I need to see if I can find any information for you to read.

Several days later Lamont came with delight on his face to tell me that our librarian had seen the questions on the board and said she had a book called *Zero is Not Nothing* (Siomer, 1978). He eagerly went to the library to bring it back.

The next week, Henry came to me and said sincerely, "You see, Ms. Willis," holding his hands closed and then opening them palms up, "zero means there's nothing. See, there's nothing in my hands. That means zero."

I was thrilled that he had actually kept this issue in his head long enough to either talk to someone about it, or come up with that explanation on his own. It was also somehow very touching to me that he seemed to be gently offering me an explanation about something I didn't yet understand.

I guess I've written up this episode for a couple of reasons. One is simply that I love the idea that even a few of my second graders can have this kind of discussion about number. This kind of "chewing" on ideas is exactly what I most hope and work for in my mathematics (actually any subject) class. I am genuinely intrigued to have this window into some second graders' thinking about what zero is. I am also thrilled that the assertion is in the air that someone invented this zero thing, as well as the particular numbers that we use. It makes them much more accessible and "touchable."

I also wonder how making sense of zero affects the children's understanding of place value. Actually, it's probably more to the point to wonder how *not* making sense of zero affects children's understanding of place value.

I am sometimes just overwhelmed with the range of ideas that bombard me in a relatively short discussion.

Student Discourse Observation Tool

Scripting of Student Discourse	Discourse Type P/F, J, or G

Student Discourse Observation Tool

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Classroom Observation - Reflection

1.
What mathematical ideas did students seem to understand? What is your evidence?

2.
With what mathematical ideas were students struggling? What is your evidence?

3.
How would you characterize the students' mathematical discourse?

Commitments

1. Read the article "3 Ways that Promote Student Reasoning."

2. Work problems 1b and 2b in the article. Mathematically justify your thinking.

3. Conduct a Structured Math Talk with your students a minimum of one time per week.

4. Bring successes and challenges to our next session.